On Possible Resolutions of the Spin Crisis in the Parton Model

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ABSTRACT: The recent EMC deep inelastic experiment using a polarized muon beam scattering on a polarized hadron target has raised serious questions about our understanding of how the spin of the proton is related to that of its partonic constituents, resulting in what has been termed a "spin crisis." Several attempts have been made to resolve the problem. We argue that none of these is acceptable. We show that for the range of Q^2 involved in the EMC experiment there exist large higher twist corrections linked to the Drell, Hearn, Gerasimov sum rule. Taking account of these effects helps, both in sign and magnitude, to resolve the problem.

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INTRODUCTION

Much excitement and bewilderment has been caused by the results of the recent European Muon Collaboration (EMC) experiment[1] on deep inelastic scattering of longitudinally polarized muons on a longitudinally polarized target, since they have been argued to imply that the total spin carried by all the quarks and antiquarks in a polarized proton is consistent with zero – a most surprising and unintuitive conclusion.

Several attempts have been made to escape this conclusion.[2,3] On the contrary protagonists of the skyrmion model[4] suggest that this conclusion is exactly what their model predicts.

In the following we comment upon and raise critical objections to the above. We also draw attention to the important role played by the Drell, Hearn, Gerasimov[5] sum rule and conclude that the total quark spin is probably not *as* small as follows from the usual analysis of the EMC result. The theoretically expected spin carried by the u and d quarks may be compatible with the data within experimental error.

CONSEQUENCES OF THE EMC DATA

What the EMC actually measure is the asymmetry

$$A \equiv \frac{d\sigma^{\substack{\rightarrow \\ \leftarrow}} - d\sigma^{\substack{\rightarrow \\ \rightarrow}}}{d\sigma^{\substack{\rightarrow \\ \leftarrow}} + d\sigma^{\substack{\rightarrow \\ \rightarrow}}} \tag{1}$$

where the top and bottom arrows indicate the muon and proton longitudinal spin directions respectively.

Strictly, A is expressed in terms of the two spin-dependent structure functions $G_{1,2}(\nu, Q^2)$ introduced by Bjorken,[6] which, in the Bjorken limit have the behavior

$$m_p^2 \nu G_1(\nu, Q^2) = g_1(x)$$

$$m_p \nu^2 G_2(\nu, Q^2) = g_2(x) \tag{2}$$

(Of course QCD scale breaking implies that $g_{1,2}(x)$ vary slowly with Q^2 and should be written $g_{1,2}(x, Q^2)$.)

The observable A actually measured is a linear combination of g_1 and g_2, but the contribution of the latter can be argued to be small and the EMC neglect it. They attempt to compensate for this in their estimate of their errors. The neglect of $g_2(x)$ is non-trivial because, in principle, $g_2(x)$ can diverge like $1/x^2$ as $x \to 0$. Nonetheless, a careful analysis[2] shows that this neglect cannot resolve the paradoxical conclusion referred to above.

Thus we accept the values of $g_1(x, Q^2)$ obtained by the EMC and in particular the crucial result that

$$\Gamma_p \equiv \int_0^1 g_1(x, Q^2)dx = 0.114 \pm 0.012 \pm 0.026. \tag{3}$$

Based upon the operator product expansion Bjorken[6] derived a sum rule for both $g_1^p(x)$ and $g_2^n(x)$. For the proton case we use a QCD corrected version of the sum rule[7]

$$\begin{aligned}\Gamma_p = \frac{1}{12} \Bigg\{ &\left[1 - \frac{\alpha_s(Q^2)}{\pi}\right] \left[a_3 + \frac{1}{\sqrt{3}} a_8\right] \\ &+ 2\sqrt{\frac{2}{3}} \left[1 - \frac{33 - 8f}{33 - 2f} \cdot \frac{\alpha_s(Q^2)}{\pi}\right] a_0 \Bigg\}\end{aligned} \tag{4}$$

where f is the number of flavours and the a_j are directly related to proton matrix elements of the nonet of axial-vector currents $A_j^\mu \equiv \bar{\psi}\gamma^\mu\gamma_5 \left(\frac{\lambda_j}{2}\right) \psi$, $j = 0, 1, \ldots, 8$, by

$$\langle p, s | A_j^\mu | p, s \rangle = 2m_p a_j S^\mu \tag{5}$$

where S^μ is the covariant spin-vector of the proton.

In the parton model the a_j are given by the following expressions:

$$a_0 = \sqrt{\frac{2}{3}} \int_0^1 dx \left\{\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}\right\} \tag{6}$$

$$a_3 = \int_0^1 dx \left\{\Delta u + \Delta\bar{u} - \Delta d - \Delta\bar{d}\right\}, \tag{7}$$

and

$$a_8 = \frac{1}{\sqrt{3}} \int_0^1 dx \left\{ \Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} - 2(\Delta s + \Delta\bar{s}) \right\} \tag{8}$$

where

$$\Delta q = q_+(x) - q_-(x)$$

and $q_\pm(x)$ are number densities for quarks with helicity parallel or antiparallel respectively to the proton's helicity. The usual $q(x)$ is the sum of q_+ and q_-.

It was shown by Bjorken[6] that the assumption of isospin invariance alone leads to

$$a_3 = \left| G_A/G_V \right| \tag{9}$$

where $G_{A,V}$ are the usual β-decay constants.

If one assumes in addition that $SU(3)_F$ is a good symmetry for describing the β-decays of the octet of hyperons, then one has[8]

$$a_8 = \frac{1}{\sqrt{3}}(3F - D) \tag{10}$$

where F, D are the axial parameters that describe all the β-decays of the baryon octet.

The value of a_0 cannot be obtained from data on β-decay. However it is directly related to the total spin carried by all the quarks and antiquarks in the proton:

$$a_0 = 2\sqrt{\frac{2}{3}}\, S_z^{\text{quarks}} \tag{11}$$

as can be seen from Eq. (6).

Using (9), (10) and (11) in (4), *i.e.* with the assumption of strict $SU(3)_F$, one can directly see[2] the problem caused by the EMC data by rewriting (4) as an equation for S_z^{quarks}:

$$S_z^{\text{quarks}} = \frac{3}{8}\left[1 - \frac{\alpha_s(Q^2)}{\pi}\left(\frac{33-8f}{33-2f}\right)\right]^{-1} \left\{ 12\Gamma_p - \right.$$
$$\left. - \left[1 - \frac{\alpha_s(Q^2)}{\pi}\right]\left[\left| G_a/G_V \right| + \frac{1}{3}(3F - D)\right] \right\} \tag{12}$$

Taking three flavours, using $|G_A/G_V| = 1.254 \pm 0.006$, and utilizing[9] $F = 0.477 \pm 0.011$, $D = 0.755 \pm 0.011$ and $\alpha_s = 0.27$ yields

$$S_z^{\text{quarks}} = 0.014 \pm 0.056 \pm 0.121 \tag{13}$$

which is compatible with zero.

ATTEMPTS TO RESOLVE THE SPIN CRISIS

Several attempts have been made to avoid this unpalatable conclusion. Close and Roberts[3] stress that the extraction of $g_1(x)$ from the data, at small x, is dependent upon the assumed behavior as $x \to 0$ of both $g_1(x)$ and the usual structure function $F_2(x)$, and that, in particular, the latter may have a more singular behavior than is given by the usual Regge analysis of small x behavior.

We are convinced that the Regge behavior

$$g_1(x) \sim x^{-\alpha_{A_1}} \tag{14}$$

where $\alpha_{A_1}(t)$ is the Regge trajectory of the A_1 meson, $\alpha_{A_1} \equiv \alpha_{A_1}(0) \simeq -0.14 \pm 0.2$, is correct, and that it is *not* possible to have a contribution to $g_1(x)$ from the $P \otimes P$ cut.[10] The reason is that only Regge poles or cuts with $G(-1)^T\sigma = -1$ can contribute to those virtual Compton scattering amplitudes that are relevant to $g_1(x)$.[11] (Here T is the t-channel isospin and σ the signature.) However, it is possible to have a three-pomeron cut, but its contribution relative to A_1-exchange is suppressed by both a factor $(m/Q)\,(ln\,\nu)^{-5}$ and a small numerical coefficient. Also we are reluctant to accept that the non-Regge singular behavior of $F_2(x)$ can be relevant at the values of Q^2 involved in the EMC experiment.

In the above we have focused attention upon the disagreement between (13) and our intuition. There are other ways of utilizing the data which suggest an actual contradiction between the data and sum rules. Let us look at the QCD-corrected parton expression for Γ_p:

$$\Gamma_p = \frac{1}{12}\left\{g_A\left(1-\frac{\alpha_s}{\pi}\right)+\frac{5}{3}g_A^s\left[1-\frac{\alpha_s}{5\pi}\left(1+4c_f\right)\right]\right.$$
$$\left.+\frac{4}{3}S_z^s\left[1+\frac{\alpha_s}{\pi}\left(1-2c_f\right)\right]\right\} \tag{15}$$

where $c_f = \frac{33-8f}{33-2f}$,

$$g_A \equiv \left| G_A/G_V \right|$$
$$g_A^s \equiv \int_0^1 dx\left[\Delta u + \Delta d + \Delta \bar{u} + \Delta \bar{d}\right] \tag{16}$$

and S_z^s is the spin carried by the strange quarks and antiquarks

$$S_z^s = \frac{1}{2}\int_0^1 dx\left[\Delta s + \Delta \bar{s}\right]. \tag{17}$$

If one now *assumes* that the spin carried by the strange sea is negligible,[8] and that $SU(3)_F$ is a good symmetry, then one can write

$$g_A^s \cong 3F - D \tag{18}$$

and (15) reduces to a QCD-corrected version of the Ellis-Jaffe sum rule.[8]

Using our previously quoted values of G_A/G_V, F and D yields a value of 0.189 ± 0.005 for the RHS of (15) to be compared with the value of Γ_p given in (3).

However, as stressed in Ref. 3, the values of F and D come from a global best fit to many hyperon decays, assuming $SU(3)_F$ symmetry. But the combination $3F - D$ can be obtained separately from several different individual or pairs of β-decay reactions, and one finds

β-decays	3F-D
np and Λp	0.390 ± 0.150
np and Σp	0.534 ± 0.086
Λp and Σp	0.498 ± 0.075
$\Xi\Lambda$	0.75 ± 0.15

to be compared with the global fit value 0.68 ± 0.02. We see that the spread of values is barely compatible with $SU(3)_F$ symmetry.

g_A^s can be estimated in an entirely different manner using the QCD sum rule approach.[12] In the latter no assumption of $SU(3)_F$ is made, but it is supposed that the strange sea is unpolarized. The result is

$$g_A^s = 0.5 \pm 0.2. \tag{19}$$

To see whether this can help let us substitute for g_A in (15) and rewrite it in the form

$$g_A^s + \frac{4 \times 1.07}{5} S_z^s = 0.139 \pm 0.09 \pm 0.20. \tag{20}$$

We see that (19) is inadequate and that (20) requires a significantly negatively polarized strange sea.

Finally we may utilize the data in yet another fashion. Substituting the parton expressions for a_0, a_3 and a_8 (Eqs. (9), (10) and (11)) and using the EMC value of (4) we can solve for the spin carried by the up, down and strange quarks separately and find[1]

$$S_z^u = 0.37 \pm 0.04 \tag{21}$$

$$S_z^d = -0.25 \pm 0.04 \tag{22}$$

$$S_z^s = -0.11 \pm 0.04 \tag{23}$$

with the total spin carried by the quarks, as already mentioned, compatible with zero.

Interestingly, Brodsky, Ellis and Karliner[4] have argued that it is natural for none of the proton spin to be carried by its quarks, to leading order in the $1/N_c$ expansion. Indeed they showed that

$$a_0 = 0$$

in the $SU(3)$, chiral invariant, Skyrme model.

An attempt to take into account chiral symmetry breaking and $SU(3)_F$ breaking leads to

$$S_z^{\text{quarks}} = -0.09$$

which is still compatible with (13). In this version one has

$$S_z^s = -0.14. \qquad (24)$$

We now come to the crucial point. We believe that neither (23) nor (24) is compatible with what is known about the strange sea.

Preparata and Soffer[13] have suggested that one can bound $|\int_0^1 \Delta s(x)\ dx|$ from our knowledge of the behavior of $s(x)$ as derived from deep inelastic neutrino experiments. There is an unjustified step in their analysis, but the idea is correct and their argument can be modified to yield a useful upper bound for $|S_z^s|$.[14] One finds

$$|S_z^s| \leq 0.036 \pm 0.015 \qquad (25)$$

which is almost a factor of four smaller than the values in (23) and (24).

We conclude that the analysis of the EMC data based upon (4), (9) and (10) (and the results of the Skyrme approach) is incompatible with the bound (25).

Recently Efremov and Teryaev[15] have offered a complete solution of the problem, with the quarks carrying about 70% of the proton's spin. Their approach is based on an interesting use of a singlet axial vector current modified by anomalous terms and which is exactly conserved. Unfortunately there is a major error in the derivation[16] and the sum rule from which their results follow is incorrect. It also seems to us that their expression for Γ_p, involving a gluonic component, is wrong.

Although outside the scope of this discussion it is only fair to mention that a calculation of $g_1(x)$ in the Massive Quark Model,[17] dating from 1985, is in rough agreement with the EMC data. However, in this approach there are no gluons and the Bjorken sum rule is not satisfied!

A NEW POSSIBILITY

We shall now show that the information contained in the Drell, Hearn, Gerasimov (DHG) sum rule[5] has a significant impact on the interpretation of the EMC results.[18]

The formulae used above include the QCD generated lnQ^2 behavior which causes scale breaking, but none of the non-perturbative or higher twist effects which must occur for small Q^2. For the usual spin-independent structure function the empirical fact of "precocious" scaling suggests that these effects are not dramatic in the region of Q^2 explored by the EMC experiment (*e.g.*, $\langle Q^2 \rangle = 4.5(\mathrm{GeV}/c)^2$ for $0.02 < x < 0.03$, $\langle Q^2 \rangle = 22.5(\mathrm{GeV}/c)^2$ for $0.3 \leq x \leq 0.4$). On the contrary for $g_1(x, Q^2)$ the DHG sum rule tells us unequivocally that there is a significant Q^2 dependence between $Q^2 = 0$ and $\langle Q^2 \rangle = 4(\mathrm{GeV}/c)^2$. In other words we must be careful to use the RHS of (4) or (15) only as the asymptotic form of $\Gamma_p(Q^2)$.

For present purposes let us ignore the small QCD correction terms and define

$$\begin{aligned} \Gamma_p^{As} &\equiv \frac{1}{12}\left[a_3 + \frac{1}{\sqrt{3}}a_8 + 2\sqrt{\frac{2}{3}}a_0\right] \\ &\equiv \frac{1}{12}\left[g_A + \frac{5}{3}g_A^s + \frac{4}{3}S_z^s\right] \end{aligned} \tag{26}$$

so that we expect

$$\Gamma_p(Q^2) \overset{Q^2 \to \infty}{\longrightarrow} \Gamma_p^{As} \tag{27}$$

Using Eq. (2) and changing integration variables from x to ν, we define*

$$I(Q^2) \equiv m_p^3 \int_{Q^2/2m_p}^{\infty} \frac{d\nu}{\nu} G_1^p(\nu, Q^2) = \frac{2m_p^2}{Q^2}\Gamma_p(Q^2) \tag{28}$$

Then for large Q^2

$$I(Q^2) \to \frac{2m_p^2}{Q^2}\, \Gamma_p^{As} \tag{29}$$

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* Note that the $G_{1,2}$ of Ref. 10 are factors m_p^4 and m_p^5 times our $G_{1,2}$ respectively.

and the RHS of (29) is empirically positive. But the DHG sum rule tells us the values of $I(Q^2)$ at $Q^2 = 0$. It is

$$I(0) = -\frac{\kappa_p^2}{4} \tag{30}$$

where κ_p is the anomalous magnetic moment of the proton ($\kappa_p = 1.79$). The crucial point is that (30) is large and negative. Hence the function $I(Q^2)$ must vary considerably, *i.e.* have important higher twist contributions between $Q^2 = 0$ and the large Q^2 regime.[19]

To get some feeling for the size of these effects we have estimated the RHS of (29) using $g_A^s = 0.5$ and $S_z^s = 0$. This is shown in Fig. 1 by the solid curve. The cross at $Q^2 = 0$ indicates the value from Eq. (30). Clearly a smooth matching of the asymptotic curve to the $Q^2 = 0$ value is not possible without large higher twist terms.

It is interesting to note that the same kind of analysis applied to the Bjorken sum rule leads to a quite different conclusion for the function

$$I_{Bj}(Q^2) \equiv m_p^3 \int_{Q^2/2m_p}^{\infty} \frac{d\nu}{\nu} \left[G_1^p(\nu, Q^2) - G_1^n(\nu, Q^2)\right] \tag{31}$$

whose asymptotic behavior is

$$I_{Bj}(Q^2) \overset{Q^2 \to \infty}{\longrightarrow} \frac{1}{3}\frac{m_p^2}{Q^2} \left| G_A/G_V \right| \tag{32}$$

and whose value at $Q^2 = 0$ is

$$I_{Bj}(0) = \frac{(\kappa_n^2 - \kappa_p^2)}{4} = 0.112. \tag{33}$$

Here the dotted curve in Fig. 1 extrapolates smoothly from large Q^2 to the value given in (33) at $Q^2 = 0$, shown by an open triangle.

We wish now to propose a quantitative estimate of the Q^2-variation of $I(Q^2)$. Any such variation must clearly be analytic, *i.e.* no singularities for $Q^2 \geq 0$. It must reproduce correctly the behavior at $Q^2 = 0$ and the asymptotic behavior as $Q^2 \to \infty$. It is evident that the minimum number of parameters needed to do this is two; one to control the value at $Q^2 = 0$, the other to set the scale for the change in sign of $I(Q^2)$. A very attractive and simple suggestion is

$$I(Q^2) = 2m_p^2 \Gamma_p^{As} \left[\frac{1}{Q^2 + \mu^2} - \frac{c\mu^2}{(Q^2 + \mu^2)^2} \right] \tag{34}$$

where

$$c = 1 + \frac{1}{8} \left(\frac{\mu^2}{m_p^2} \right) \frac{\kappa_p^2}{\Gamma_p^{As}} \tag{35}$$

and μ is a mass-parameter setting the scale for the Q^2 variation. The data of Ref. 19 suggests that $\mu \lesssim m_\rho$ is required.

The form (34) with $\mu^2 \approx m_\rho^2$ has some motivation from the point of view of the vector dominance model (VDM). The second term represents diagrams where both virtual photons interact with the nucleon via vector mesons. Its contribution ought then to be negative at small Q^2 if the VDM correctly describes nucleon Compton scattering. The first term in the RHS of (34) can be interpreted as representing the diagram where only one virtual photon interacts via a vector meson and the other directly with the hadron (or via some set of heavy excited states).

Using again $g_A^s = 0.5$, $S_z^s = 0$, we plot $I(Q^2)$ as given by (34) in Fig. 2 for $\mu = m_\rho$. Also shown is the asymptotic form given in (29).

We must now face the question as to whether our Q^2-dependence is compatible with the results of the EMC experiment. They studied the Q^2-dependence of their data in fixed-x bins and concluded that they had no evidence for any variation with Q^2. However the errors in these fixed-x plots are large and we estimate that the data are also compatible with a deviation of about 20% at $Q^2 = 10(\mathrm{GeV}/c)^2$ and 30% at $Q^2 = 5(\mathrm{GeV}/c)^2$ between $I(Q^2)$ and its asymptotic form. We therefore believe that our $I(Q^2)$ with $\mu \sim m_\rho$ is compatible with the EMC data and may be used as a reliable estimate of the higher twist effects.

Finally then, let us put $\mu = m_p$, take $Q^2 = 10(\mathrm{GeV}/c)^2$, which corresponds to the mean value in the EMC experiment, and determine Γ_p^{As} by demanding that

$$\Gamma_p(Q^2 = 10) = \left[\frac{Q^2}{2m_p^2} I(Q^2)\right]_{Q^2=10} = \Gamma_p^{EMC} \tag{36}$$

We find

$$\Gamma_p^{As} = 0.144 \pm 0.013 \pm 0.029 \tag{37}$$

which is about 30% larger than Γ_p^{EMC}.

Using the value (37) for Γ_p, Eq. (20) is modified to

$$g_A^s + \frac{4 \times 1.07}{5} S_z^s = 0.364 \pm 0.098 \pm 0.218. \tag{38}$$

We see that with the higher twist corrections the left and right hand sides of (38) are compatible, within experimental error, with the QCD sum rule value $g_A^s = 0.5 \pm 0.2$ and a small S_z^s contribution.

Alternatively, using the value (37) for Γ_p in Eq. (12) we obtain respectively for the case of global or QCD sum rule values of $3F - D$,

$$S_z^{\text{quarks}} = \begin{cases} 0.093 \pm 0.064 \pm 0.133 & (\text{global } 3F - D) \\ 0.115 \pm 0.064 \pm 0.133 & (\text{QCD sum rule}) \end{cases} \tag{39}$$

Bearing in mind the experimental errors, we see, especially for the QCD sum rule value, that the spin carried by the quarks, which is many times bigger than the value quoted in (13), is now much more reasonable and is compatible with 50–60% of the total spin of the proton.

From (15) we now find for the spin carried by the strange sea

$$S_z^s = \begin{cases} -0.082 \pm 0.040 \pm 0.055 & (\text{global } 3F - D) \\ -0.045 \pm 0.040 \pm 0.055 & (\text{QCD sum rule}) \end{cases} \tag{40}$$

These values are compatible with the upper bound (25).

We conclude that the higher twist effects associated with the DHG sum rule are very important.

We note that a higher value of the parameter μ would lead to a larger value of Γ_p^{As}. Taking, for example, $\mu = 1\,\mathrm{GeV}/c^2$ gives $\Gamma_p^{As} = 0.17$. Such a value, however, leads to $I(Q^2) = 0$ at $Q^2 \simeq 2.6(\mathrm{GeV})^2$ and to strong Q^2 dependences which do not seem to have been observed. Further measurements in the low Q^2 region would decide whether this is acceptable.

CONCLUSIONS

We have shown that none of the earlier proposed solutions to the "spin crisis" are satisfactory. Their effects are either too small or they are in contradiction with the bound on the strange sea contribution.

We have demonstrated, for the range of Q^2 involved in the EMC experiment, that there must be large higher twist corrections connected to the Drell, Hearn, Gerasimov sum rule. By taking account of these quantitatively we achieve a solution which is compatible with the quarks carrying 50–60% of the proton's spin and which is in accord with the bound on the strange sea contribution.

It is evident that confirmation of the EMC result is a vital matter. Equally, that it is essential to measure $g_1(x)$ for the neutron. A failure of the Bjorken sum rule would imply that QCD is wrong! It is particularly important to make a careful study of the Q^2-variation at small x.

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FIGURE CAPTIONS

Figure 1: Comparison of the asymptotic forms of $I(Q^2)$ for G_1^p (proton sum rule) and $G_1^p - G_1^n$ (Bjorken sum rule) with the corresponding Drell-Hearn-Gerasimov values at $Q^2 = 0$ ($\times$ and Δ respectively).

Figure 2: Comparison of the asymptotic form of $I(Q^2)$ for G_1^p with our Eq. (34) for $\mu = m_\rho$ and $\mu = 1\,\mathrm{GeV}/c^2$.

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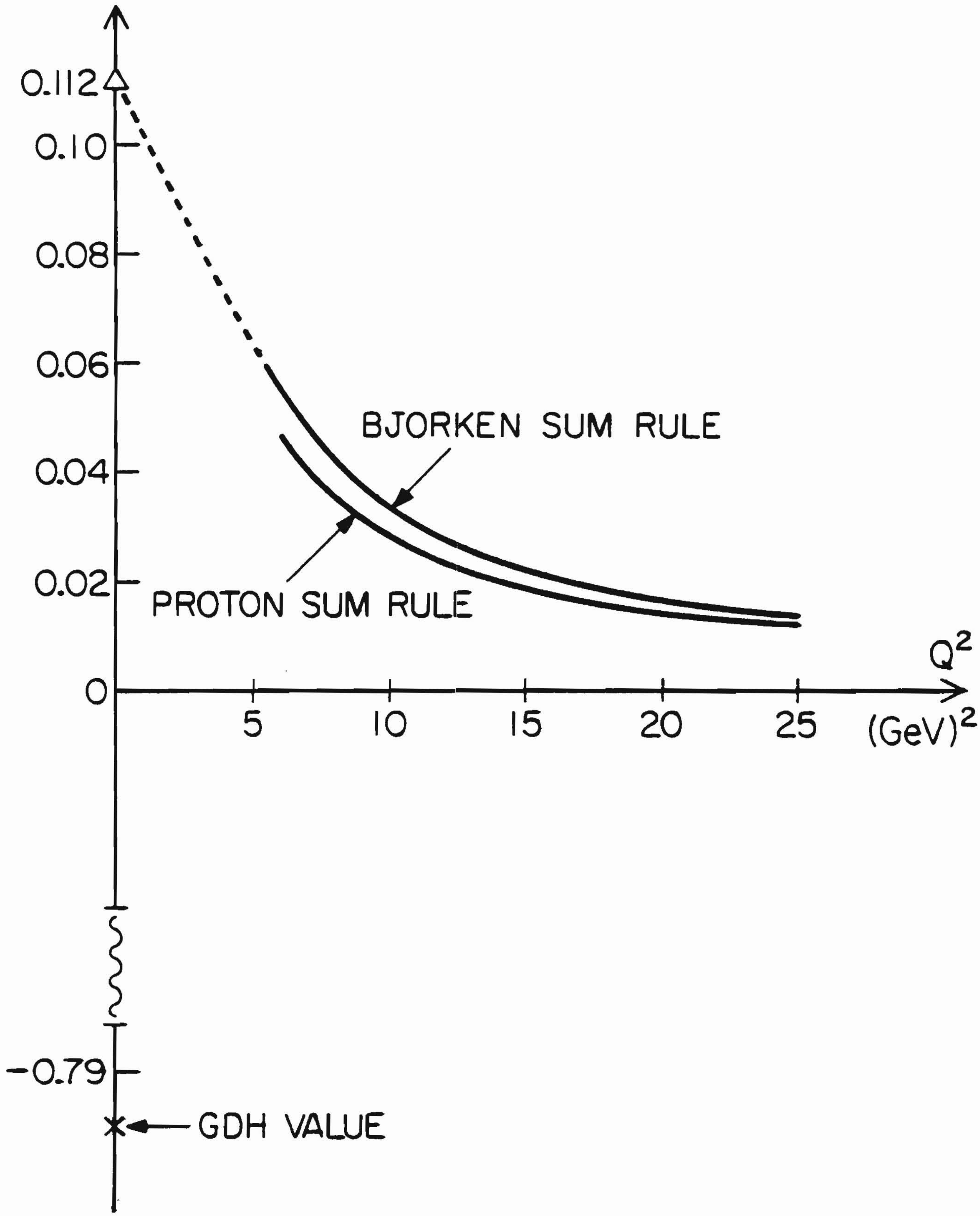
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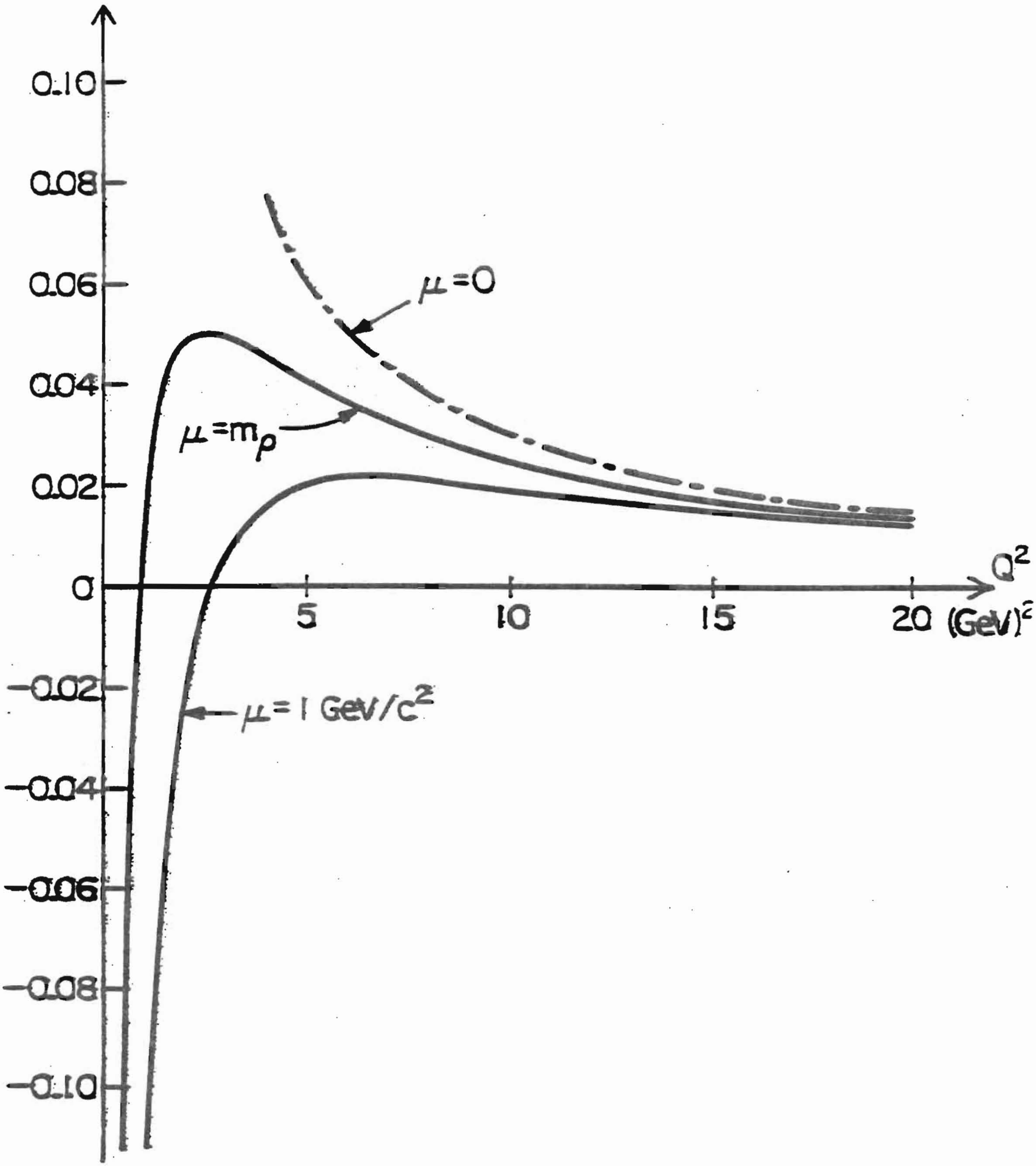
Figure 1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-0.02
-0.04
-0.06
-0.08
-0.10
μ=0
μ=m_ρ
μ=1 GeV/c²
Q²
5
10
15
20 (GeV)²

Figure 2